A Dynamic Model

of Internally-Driven Corporate Social Responsibility

and Enlightened Profit Maximization

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Abstract

The aim of this paper is to contribute to the analysis of Corporate Social Responsibility (CSR) from an economic perspective, in two ways. Firstly, introducing a new definition and a new framework of analysis, which can account for both the *externally-driven* and the *internally-driven* view of CSR. Secondly, developing a dynamic model of *internally-driven* CSR – which draws inspiration from the literature on renewable resources – to show that, under certain circumstances, an *enlightened* profit-maximizing firm will behave as 'Socially-Responsible' to the eyes of the society. The paper also discusses some possible limits and extensions of this theoretical framework.

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When an optimising entity has a sufficiently encompassing interest on the society […] the same self-interest that leads him to maximize his extraction from the society also gives him an interest in the productivity of his society"

McGuire and Olson (1996), p. 76

1. Introduction

The issue of Corporate Social Responsibility (CSR) has gained increasing importance over the past years. While definitions vary, there is a general consensus on the fact that CSR includes all "*situations where the firm goes beyond compliance and engages in actions that appear to further some social good, beyond the interests of the firm and that which is required by law*" (McWilliams, Siegel and Wright, 2006).

However, the idea that a firm incurs responsibilities towards the society beyond its own interest, is not exempt from critique. Perhaps the main counter-argument is still the one made by Milton Friedman (1970), who claimed that, from an economic perspective, it would be bad to shift resources from firm's the primary objective of maximizing profits. In fact, from a neoclassical perspective, maximising profits implies an efficient allocation of the resources, and hence maximises social welfare.

Recent empirical studies reconcile these two views, showing that in many occasions the companies which implemented CSR practices to "*further some social good*" (see above), also experienced a wide range of bottom-line benefits. These include - among many others - enhanced brand image and reputation, reduction of waste and introduction of more efficient environmentally-friendly production techniques, increased work productivity, greater ability to attract and retain employees, reduced costs from injuries and absenteeism, reduced regulatory oversight, increased support by the neighbouring communities, improved access to capital, government licenses and global supply-chains.[1]

[1] The companies themselves are also increasingly aware of this relationship between CSR and profits. For example, the 2008 Annual Report of ArcelorMittal (p. 27) states: "It is ArcelorMittal's conviction that business growth, sustainable communities and the creation of shareholder value go hand-in-hand. Only by addressing the global issues affecting its business, its people and its communities, can ArcelorMittal help establish mutually beneficial stakeholder relationship, attract and retail top talent and maintain the *license to operate* […] achieve high impact Health and Safety improvements that protect the company's greatest assets – our people: the Lost Time Injury Frequency Rate improved in 2008 to 2.3 compared to 3.3. in 2007."

In light of this, concepts such as 'Business case for CSR' and 'Strategic CSR' are now widely used in the management literature, to identify business practices which yield benefits for the environment and the society at large, and also private benefits for the firms (see Porter and Kramer, 2002). From this perspective, 'Strategic CSR' is a distinct phenomenon than traditional charity or philanthropy, from which firms do not expect any return.[2]

In our paper we focus exclusively on this type of CSR, in order to eschew the domain in which Friedman's critique would apply, which is every 'non-business' argument for CSR, as we shall see later in the paper.[3] All the ethical considerations, which could arise on certain aspects of 'Strategic CSR', go beyond the scope of economic analysis.

Among the many arguments which have been proposed, to explain why firms engage in 'Strategic CSR', we introduce here a distinction between those which see CSR as '*externally-driven'* and those which see it as *'internally-driven'*. According to the *externally-driven* view, there is a demand for a more ethical behaviour by the firms among one or more of the stakeholders (e.g. consumers, workers, investors, government, etc.), which leads the firms to undertake CSR as a differentiation strategy. If this is the case, the CSR practices undertaken should reflect stakeholders' preferences.[4] On the contrary, *internally-driven* CSR is a strategy undertaken by the firms to manage optimally the factors of production, with the objective of maximising long-term profits. Hence, the practices adopted should be closely linked to the firms' core business activities.

The remainder of the paper is structured as follows. Section two presents a short review of the economic literature on CSR, claiming that so far only *externally-driven* views have been taken into consideration. Section three introduces a new definition and a new framework of analysis for CSR, which can account for both *externally-driven* and *internally-driven* views. Section four presents

[2] In the literature it is now more and more common to refer to CSR as only the 'Strategic' component of CSR, while the rest is called philanthropy or charity in general. For example, Antonio Gaspar (2003, p.3) defines CSR as "an investment from which companies should expect tangible returns and positive impact on their net profits", while "philanthropy relates to donations or charitable giving from which companies do not necessarily expect any direct positive impacts on their business activities."

[3] Another reason for focusing on 'Strategic CSR', rather than philanthropy in general, is that many empirical studies showed that the former is also more effective from a social welfare perspective. For example, a study by UNDP (2005) showed that only the projects which are sufficiently driven by business profitability can be considered sustainable in the long run, while charitable contributions depend too heavily on available cash-flows and therefore are often only had-hoc interventions.

[4] Prof. Jean Tirole, speaking at the third annual *Economica*-Coase lecture at the LSE on 19th February 2009, on the subject "Individual and Corporate Social Responsibility", classified as 'Delegated Philanthropy' all the CSR practices in which companies "act on behalf of their stakeholders". This is very similar to our definition of *externally-driven* CSR.

a dynamic model of *internally-driven* CSR and, finally, section five concludes the paper, discussing some limits and possible extensions of our approach.

2. A short review of the economic literature of CSR.

To the best of our knowledge, all the theoretical models of CSR which have been developed so far view CSR as being exclusively *externally-driven*. One of the earliest papers to introduce this view of CSR into the classical economic framework of supply and demand, is Baron (2001). In this paper, it is assumed that consumers are willing to pay a higher price for the products sold by firms which undertake activities to protect the environment. Since, at the beginning, the costumers value an improvement in the environment – in terms of an additional willingness to pay for the product - more than what it costs for the firms to provide it, the firms invest in improving the environment until they reach the level at which marginal benefits equal marginal costs.

However, Baron argues that his model can only explain one bit of the CSR undertaken by firms – the 'strategic' CSR - while the other - the 'true' CSR – should go beyond this profit-maximising level, reflecting the altruistic preferences of firms' owners or shareholders. This argument presents one intrinsic problem, which is that it does not distinguish between social and private costs vs. benefits from an action. In fact, "as long as there are no negative externalities, a firm should expand its output until the value to society of the goods and services produced is at least as great as the price the firm receives" (Jensen, 2001, p.303). This implies that the profit-maximizing level of CSR is also the socially optimum level, and any additional unit of CSR beyond this point would decrease overall welfare. This is also the cornerstone of Friedman's (1970) argument, and is the main reason why, as mentioned in the introduction, our paper will focus only on 'Strategic CSR'.

One of the most influential papers which analysed 'Strategic CSR' from an economic perspective is McWilliams and Siegel (2002), in which the authors analyse the different impact of CSR on firms' interaction in the market, depending on the products' characteristics and the types of competition. This contribution set the stage for more formalized models of *externally-driven* CSR, which can be classified into the following categories, according to the way the authors define CSR: (i) as the private provision of a local public good/reduction of a public bad, from which consumers gain a certain utility (Bagnoli and Watts, 2003; Besley and Gathak, 2007); (ii) as a differentiation strategy,

to exploit the increasing sophistication of consumers' demand (Becchetti, Giallonardo and Tessitore, 2005; Becchetti, Federico and Solferino, 2005; Manasakis, Mitrokostas and Petrakis, 2007; Evangelios and Petrakis, 2007); (iii) as a labour market screening strategy, to attract the most motivated and productive employees (Brekke and Nyborg, 2005); and, finally, (iv) as a strategy to avoid increasing government regulation (Maxwell, Lyon and Hackett, 2000; Baron, 2001).[5]

Despite the numerous theoretical insights, there are some common shortcomings in these models, which derive from the fact that they view CSR as being exclusively *externally-driven*. First of all, in these models CSR always occurs at a cost for the firms, which, as mentioned in the introduction, is not always the case. Secondly, as it has been addressed when discussing the paper by Baron (2001) – this view fails to distinguish between social and private costs vs. benefits. Finally, there are two additional major problems: asymmetry of information and free riding.

The problem of asymmetric of information arises because CSR is a 'credence good' (Manasakis, Mitrokostas and Petrakis, 2007), which means that consumers' willingness to pay is determined by their beliefs over the ethical attributes of a product, but that they cannot infer anything about these attributes simply by buying it, or consuming it. As a consequence, a time-consistency problem arises: the firms have no incentive to comply with their promises once the consumers believe to them, and the consumers anticipate this incentive and do not pay the higher price for allegedly ethical products. As a result, firms will not undertake any CSR.[6]

The solutions to this problem, which have been proposed in the literature, are to introduce (i) a monitoring technology, which the firms themselves would be willing to pay in order to make their promises more credible (Besley and Gathak, 2007); (ii) a market for socially responsible managers, sustained by the mechanism of reputation, where firms could hire to signal their commitment to CSR (Manasakis, Mitrokostas and Petrakis, 2007); and, finally, (iii) a government certification for CSR (Mitrokostas and Petrakis, 2007). These and other similar solutions, which rely mainly on the role of Civil Society Organizations as monitors, have worked well in reality to reduce the extent of the asymmetry of information between the firms and the stakeholders.

[5] It might not be evident why the latter belongs to an *externally-driven* view of CSR. However one has to consider that the demand for government regulations – which the firms are trying to pre-empt – often comes from the Civil Society and other lobby groups, which are indirectly a stakeholder of the firm. This will be clarified in the next section of the paper.

[6] This theoretical problem has a great empirical relevance: in fact, it is quite common to hear news from companies claiming to undertake certain CSR practices, which then proved not to be true.

The problem of free riding has been highlighted by Bagnoli and Watts (2003) and Besley and Gathak (2006), in the context of CSR as private provision of local public goods. According to them, CSR is subject to the same free-riding problem as any voluntary contribution to public goods. In fact, to provide the public good firms have to charge a higher price on their products, however only some of the consumers, which would benefit from the public good, actually pay this higher price. Therefore, these firms do not capture all the common benefits from the provision of the public good and will provide less than what it would be socially optimal.[7] This argument holds even if consumers' preferences exhibit a 'warm glow' component (Andreoni, 1989)[8] and can be extended to all *externally-driven* models of CSR.[9] Though, a convincing solution to it has not been found yet in the CSR literature.

In conclusion, there seem to be some intrinsic limits in analysing CSR exclusively as *externally-driven*, which makes it necessary to integrate into the framework of analysis some aspects drawn from the *internally-driven* view. This is the objective of the following section.

3. A new framework of analysis

First of all, we define CSR as "*every activity that a firm undertakes voluntarily, based on a sensible economic incentive, resulting in the full or partial internalisation of the externalities – positive and negative – on the society and the environment associated with the firm's production of goods and services".*

This definition takes into account that CSR is a voluntary behaviour, and it has the advantage of relaxing the assumption that CSR has to be strictly profit-enhancing, and also of capturing the distinction between private and social benefits vs. costs of production. In fact, in presence of a negative externality, the profit-maximising production level of the firm's goods or services is too

[7] The socially-optimal level of CSR would be given by the standard Lindahl-Samuelson rule of marginal costs equal to the sum of marginal benefits. For completeness, in the model by Bagnoli and Watts (2003), there are some cases in which there is actually overprovision of the local public good.

[8] A 'warm-glow' component means that consumers receive not only the direct utility from the public good, but also an indirect utility from having contributed to it, which comes from altruistic preferences or social status considerations. Besley and Gathak (2006) showed that the presence of this component among some of the consumers, can mitigate the problem of free riding, but not solve it completely, unless it is strong enough to compensate for the free-riding of all the others.

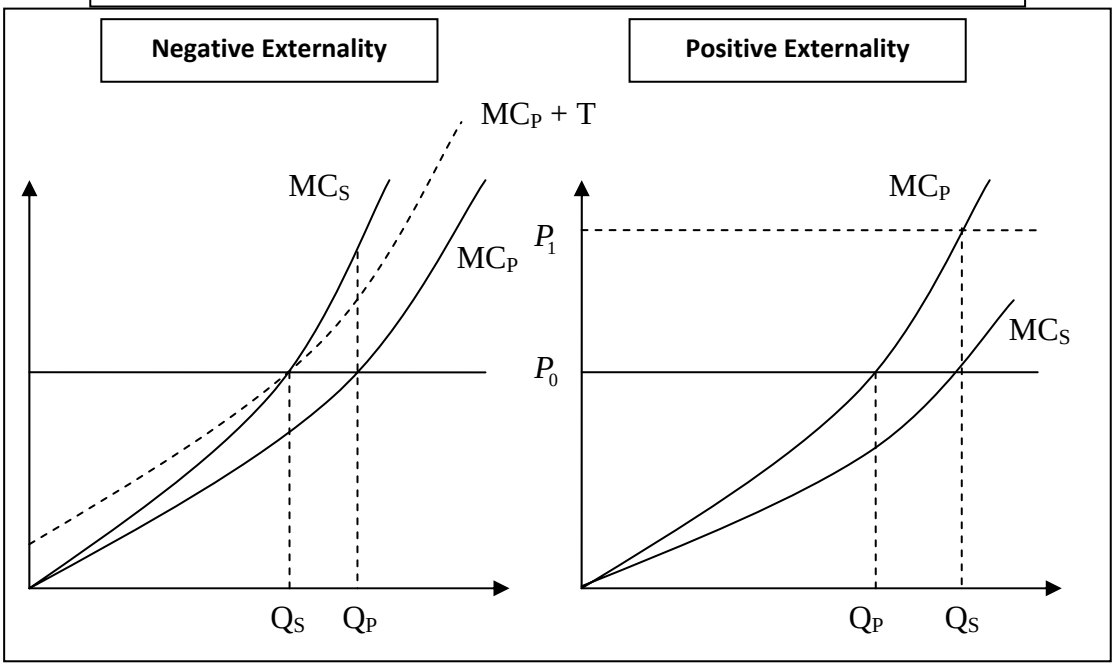
[9] For example, Brekke and Nyborg (2001) show that 'motivated' workers provide less effort than what they would themselves consider morally best. In the authors' words, a worker, despite "stretching towards his conception of morally ideal behaviour, he stops short of reaching that ideal" (ibid., p. 12).

high with respect to the social optimum, and the opposite occurs when there is a positive externality. The key is now to understand why a profit-maximizer firm would voluntarily choose to internalise these externalities - i.e. provide more of the 'public good' and less of the 'public bad' associated with its activities.[10]

The classical mechanisms by which firms can be induced to internalise some of the negative externalities they produce is through a *Pigouvian Tax*. Of course, this would not be a voluntary intervention, but the same result can be achieved if firms decide to pre-empt the introduction of such a tax, by voluntarily limiting the production or implementing measures to reduce the level of the externality (cf. the case of pollution).[11]

In the opposite case of a positive externality, one way in which firms would be induced to provide more of the 'public good', is if consumers were willing to pay a higher price for the private good from which the positive externality arises (see Figure 1).

Figure 1. Traditional solutions to the problem of externalities



[10] One way to achieve this result is to assume, as Kelsey and Milne (2006) do, that the firm's shareholders are also the people which are directly affected by the externality. In this case, the incentive for internalising them is straightforward, but it does not explain why firms would still behave like that in more realistic situations in which the shareholders are not directly affected by the externality.

[11] The economic incentive to do so is given by the fact that any voluntary reduction of the level of production (or reduction of the 'public bad' associated with the production), which is strictly less that what the tax-regime would lead the firm to do, leaves the firm better-off with respect to the situation with the tax.

These two cases are characteristic of *externally driven* views of CSR, hence our externality framework accounts well for these. Let's consider now how it can be applied to *internally-driven* CSR.

In order to do so, we start from the consideration that firms need several factors of production, of which not only the level is important, but also the quality. In the introductory section, we have seen the CSR can affect the availability and the level of some factors of production (e.g. water, agricultural crops, forests, etc.), and the quality of others (e.g. labour productivity - where elements such as employees' motivation, satisfaction and well-being play a key role - support of the neighbouring communities, access to government licenses and foreign technology, etc.). [12]

These factors of production also have a value for the society as a whole, i.e. they are 'public goods', in the sense that everyone in the society benefits from a richer environment, a higher level of satisfaction and happiness of employed people and their families, and so on.

Therefore, a potential conflict could arise between the needs of firms - which want to use these factors to produce - and the ones of the society - which would like to see them grow. This conflict falls nicely into our externality framework: by employing one factor – and thus reducing the available stock - a firm imposes a negative externality on the society, while by investing into increasing its stock, it generates a positive one.

This is the starting point of our analysis of *internally-driven* CSR, which identifies all the situations in which the firm's choices towards its factors of production are not in conflict with the interests of the society, but actually aligned with them.

To develop the theoretical model, we draw inspiration from the literature on renewable resources. This literature studies the optimal rate of exploitation of a scarce resource (e.g. fisheries), which would grow over time if unexploited, but could be exhausted if excessively harvested (Dasgupta and Heal, 1979; Clark, 1990). These features are also relevant for the factors of production on which CSR has an impact.

First of all, it seems plausible to assume that the stock of these factors grows over time. In some cases it is the level that grows (e.g. agricultural crops, forests, natural resources, etc.), in others the quality (e.g. labour productivity, support from neighbouring communities, etc.). In the – maybe

[12] One clarification is necessary here. What the firm pays for is always for the level of a tangible factors of production (e.g. materials, or number of employees), but when CSR can affect the quality of this factors, then the firm can obtain from it a marginal product which is higher than its actual marginal cost.

less evident - case of quality, this is due to the way reputation spreads over time and to the formation of habits among the stakeholders (Becchetti, Giallonardo and Tessitore, 2005). This implies that the quality of the factors of production will increase over time, and this applies both to the factors that a firm could decide to use in the future (e.g. the more widespread a firm's reputation, the higher will be its ability to attract the most motivated employees), and to those which the firm is already employing (e.g. the longer an employee has been working for a firm, the higher will be his/her motivations, because of increasing loyalty and self-identification; similarly, or the longer a firm has settled in a community – and the more of its members found a job in the firm - the greater will be the support from the community).

Secondly, when firms use these factors of production, they reduce the stock available for future production – again, either the level, the quality, or both. The Civil Economy literature (Bruni, Zamagni, 2004) helps us understand how this process might work in the case of quality. According to this literature, economic agents receive a greater utility in environments where a lot of 'relational goods' are provided. These are goods produced by the interaction among agents, such as friendship, trust and social reputation. Translating this idea into our model, we can argue that relational goods are becoming more and more abundant in private firms, thanks to their engagement in CSR. This determines that, for example, there are workers who are willing to give up a higher wage to work for firms which they perceive to be more ethical (Collier and Esteban, 2007; Brekke and Nyborg, 2005), or investors willing to give up higher returns to invest in ethical funds (Baron, 2001). If this is the case, then a firm which 'owns' a certain level of reputation coming from its past behaviour, could decide to exploit it to obtain an advantage - for example making motivated employees work longer hours, or using its past CSR achievements to acquire a licence to operate. However, this will reduce the stock of relational goods, because of increasing dissatisfaction and lack of trust, unless the firm does something to build up again the stock of relational good. Hence, the key question that our model will try to answer is how much a firm should exploit the benefits of its reputation today, and how much it should leave 'unexploited' - or even invest into increasing the stock of relational goods – to obtain a higher quality of the factors of production in the future.

4. The Model

A firm *i* employs a certain factor of production y to produce an output z, which then sells at a constant price $\overline{p}$ (i.e. there is perfect competition in the market of the final output). For simplicity, and without loss of generality, let's assume the simplest production function possible, $z_t = ky_t$, where *k* represent the overall efficiency of the production process.

The factor of production is available in a certain stock, *x*, which, as previously noted, is determined both by its level and its quality. The firm, however, cannot employ the factor directly, paying a price for it, but has to employ an 'instrumental input', E, at a unitary cost *w*, to obtain it. Then, the amount of the factor which will actually enter in the production process, is an – increasing - function of the amount of the instrumental input employed, and also of the level of the stock available: $y_t = h(E_t, x_t)$.[13]

This formulation is quite flexible and can be applied to all the factors of production mentioned in the previous section. For example, in the case of a natural resource, *w* can be seen as the unitary cost of a machinery to extract the resource and E as the number of machineries. The amount of the factor of production which will actually enter in the production process is a function of the number of machineries, but also of the stock of the factor. In fact, when this is available in great quantity, the firm will be able to obtain more of it for a given number of machineries.

In the case of human capital, *w* can be viewed as the salary paid to the employees, while E is the number of employees. Again, there is not a one-to-one correspondence between the number of employees and the level of human capital which enters in the production process, because the employees might decide to put more or less effort at work. This decision depends on their 'stock' of motivations, which in turn depends on the firm's reputation, goodwill etc.

An important element to be noted - and which distinguish our *internally-driven* view of CSR from the *externally-driven* presented before - is that this level of motivations is not exogenous and taken as given by the firm, but is endogenous to our model, depending on the firm's behaviour.

Following Schaefer (1957), we assume the simplest functional form to capture how an additional unit of the instrumental input E translates into additional units of inputs *y*, given by:

[13] We maintain the notation typical of the literature on exhaustible resources, where E is the level of the *effort* (to extract a certain resource) and *h* is the *harvesting* function.

$$y_t = E_t x_t \tag{1}$$

We start by analysing the behaviour of a profit-maximising firm, which does not take into account the effect of its activities on the level and the quality of the factors of production. The firms' total profits are given by the stream of present and future profits, discounted at a rate equal to ρ:[14]

$$\prod = \int_0^\infty e^{-\rho t} \left(\bar{p} k y_t - w E_t \right) dt \tag{2}$$

Plugging equation (1) in the equation of the profits, we obtain the following expression:

$$\prod = \int_0^\infty e^{-\rho t} \left(\bar{p} k E_t x_t - w E_t \right) dt \tag{3}$$

The profit function is linear in the control variable - the level of the instrumental input – hence we have that if, at any time *t,* $\bar{p} k x_t < w$, the firm would not produce at all, while if $\bar{p} k x_t > w$ it would want to produce an infinite amount of the final good, and thus to employ an infinite amount of the factor of production. In this case however - even if the firm does not take that into account – the fact that the firm uses the factor will reduce the stock *x* available in the next period. Hence, as production goes on, the firm will obtain less and less additional units out of a given level of the instrumental input, and this process will continue until $\bar{p} k x = w$. At this point the firm will have to stop to employ the factor, because obtaining one additional unit would yield negative profits ($\bar{p} k x < w$). Following this reasoning, at equilibrium we must have that:

$$\bar{p} = \frac{w}{kx} \tag{4}$$

which states that that the marginal revenue of employing one additional unit of the factor of production – given by the (constant) price $\bar{p}$ of the final good multiplied by the efficiency of production parameter *k* - has to be equal to the marginal cost of obtaining this additional unit - given by the marginal cost of the instrumental input, divided by the additional units of the factor of production that the instrumental input will allow to obtain (i.e. $\frac{\partial h(E,x)}{\partial E} = x$).

Now we analyse how differently an '*enlightened*' firm would behave. By *enlightened* firm, we mean a profit-maximising firm which takes into account the 'dynamic stock externalities' from its

[14] The firm's time-discount rate is often assumed to be equal to the interest rate, because a firm can always reinvest their profits at the interest rate. However we will see later on in the paper how we can use it to capture the relative weight which firms put on 'short-term' profit maximization vs. 'long-term' profit maximization.

production, i.e. how its production choices affect the stock of the factors of production available in the future.

First we need to define how the stock of the factor evolves over time. For the purposes of this paper, we will not use any specific functional form, but only assume that the growth of the factor of production is a function of the stock, $\frac{dx}{dt} = F(x)$, and that this growth-function satisfies some key proprieties: $F'(x) > 0$, $F''(x) < 0$ and $\lim_{x \to 0} F'(x) = \infty$.[15]

Therefore, the actual change in the stock of the factor of production is given by this growth-function, minus the amount actually employed by the firm:

$$\dot{x} = F(x) - h(x, E) \tag{5}$$

Using dynamic programming, we can solve the problem of maximising the infinite stream of profits, given by equation (3), subject to the law of motion in equation (5).

Proposition 1. An *enlightened* firm – defined as a firm which takes into account how it affects the level and the quality of certain factors of production (i.e. the *dynamic stock externalities*) - will stop employing these factors, before reaching the level at which the marginal revenues from using them equal the marginal costs of obtaining them.

Proposition 2. This distance, between the level of the factors of production employed by an *enlightened* firm, and the level at which marginal revenues equals marginal costs, decreases in the firm's time-discount rate (i.e. it becomes smaller, the more weight the firm puts on maximizing 'short-term' profits). In the limit, a firm with an infinite discount rate (i.e. which does not care at all about future profits), will employ the factors of production until marginal costs equal marginal revenues, 'as if' it did not take into account the *dynamic stock externalities*.

[15] In the literature on renewable resources, the most widely used function to define how a population grows over time is Logistic growth function, which has the following form: $\dot{x} = rx\left(1 - \frac{x}{K}\right) = F(x)$, where K is referred to as the system's *carrying capacity*, or *saturation level* (Clark, 1990). Despite not being used in our paper to avoid any loss of generality, the implications of using this particular functional form in terms of our model would be the same. The mathematical derivation is available upon request.

Proof. *The Current Value Hamiltonian for this problem is:*

$$H = \overline{p}kEx - wE + \lambda\left[F(x) - Ex\right] \tag{6}$$

Deriving the first order conditions and cross-substituting,[16] *we obtain our expression for the optimal exploitation of the resource at every time:*

$$\rho = F'(x) + \frac{\frac{w}{x^2}F(x)}{\overline{p}k - \frac{w}{x}} \tag{7}$$

where the first derivative of the factor of production's growth function, $F'(x)$*, is referred to as the 'own/internal rate of return' in the literature on renewable resources.*

The formula implies that a firm should increase the level of the instrumental input, up to the point at which the time-discount rate is equal to the factor of production's 'own rate of return', plus an additional element which captures the reduction in future costs for obtaining the factor. In fact, renouncing to obtain the resource today will increase the future stock, and – because of $\frac{\partial h(E,x)}{\partial x} > 0$ *- reduce future costs.*

To prove Proposition 1 and 2, first we make a no-loss assumption, i.e. $\overline{p}k$ *has to be always greater or equal than* $\frac{w}{x}$*, or the firm would make negative profits out of each unit of the factor employed. Then, we analyse what happens when the stock of the factor of production is very small, and when it is very large.*

For values of the x *small enough, the 'own rate of return' has to be greater than the firm's discount rate,* $F'(x) > \rho$*, and in this case the firm will not employ the factor of production at all, because it will make higher profits by letting the stock grow and use it in the future.*[17]

[16] See the Appendix for complete mathematical derivation.

[17] Note that - for the non-negativity of the profits assumption - the second term on the left-hand side of equation (7) has to be positive, hence the whole left hand side of the equation will be greater than ρ. For this result to be realistic, however, we will need to make some assumptions on the fact that (i) if the firm does not utilises the factor, someone else will not do it, and also that (ii) if it does not employs it today, it will be able to do so in the future. These aspects are discussed in Preposition 4, in the last section of the paper.

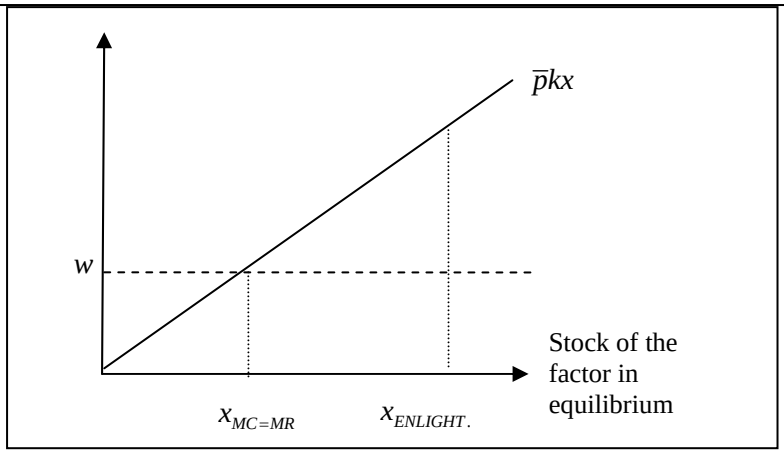
For values of x large enough, ρ will be greater than the right-hand side of equation (7)*, since we know that, as x increases,* $F'(x)$ *decreases, while* $\lim_{x\to\infty}\frac{\frac{w}{x^2}F(x)}{\bar{p}k-\frac{w}{x}}=0$*. In this case the firm will find it profitable to employ the instrumental input to obtain the factor of production, and will produce the final good. As this process goes on, the stock of the factor will be reduced,* $F'(x)$ *will increase - because of diminishing marginal returns – and* $\frac{\frac{w}{x^2}F(x)}{\bar{p}k-\frac{w}{x}}$ *will also increase, for the admissible range of the parameters.*[18] *Therefore, the right-hand side of equation* (7) *will increase until it will be equal to* ρ*. At this point the firm will stop employing the factor of production.*

During the period in which the firm uses the factor, the difference between the marginal revenues from employing one additional unit of the factor, $\bar{p}k$*, and the marginal costs of obtaining it,* $\frac{w}{x}$*, will decrease. It is evident that the equality in* (7) *will be verified before the level at which* $\bar{p}k=\frac{w}{x}$*, when the term* $\frac{\frac{w}{x^2}F(x)}{\bar{p}k-\frac{w}{x}}$ *would be infinite. Hence, an enlightened firm will stop employing the factor, before it reaches the level at which marginal revenues equal marginal costs, and the level of the stock in equilibrium will be higher (see Figure 2).*

From the formula it is also evident that, the greater ρ*, the further the firm will go in employing the factor, until for* $\rho\to\infty$ *the level of utilization will be equal to the one at which* $\bar{p}k=\frac{w}{x}$*.* ***QED***

[18] In particular, the non-negativity of the profits has to hold. In fact, the derivative of this term with respect to x, is negative if $2\bar{p}kx>w$. Hence, the non-negativity of profit assumption is a sufficient condition to ensure that, as x decreases, the term into consideration will increase.

Figure 2. 'Enlightened' firms and level of employment of the factors of production



So far CSR has been introduced in our model only in an indirect way. However, we can think of CSR entering in a more explicit way at different stages of the model, affecting, for example (i) the rate of growth of the resource over time, or (ii) the overall efficiency of the production process.

The first case includes all CSR programmes which increase the level of a factor of production (e.g. reforestation programmes, more sustainable production processes etc.), or its quality (e.g. flexitime, work-life balance, childcare facilities, trainings, etc.). A growth function which captures these elements could look like this:

$$\frac{dx}{dt} = F\left(x, S\right) \tag{8}$$

where the level of CSR, S, increases the marginal rate of growth of the factor of production. The existing literature on CSR helps us characterize some of the key proprieties of this function. For example, the benefits of CSR are usually very high at the beginning, because the firm can address the 'low-hanging fruits' (Husted and Salazar, 2006), and then decrease over time, until they reach a point – which we call S_{MAX} - in which an additional unit of CSR does not yield any additional benefit for the resource (McWilliams and Siegel, 2002).[19] To reflect these elements, we shall assume that

[19] It is important to note that these benefits are not private benefits for the firm, as for example in Baron (2001), but are benefits for the resource itself, in terms of increased availability and quality. Then, because in our model we have that the firms and the society as a whole put a value on the stock of these factors of production, both of them will benefit from the increased stock. Hence, we do not have the problem of distinguishing between private vs. social benefits, as it was for *externally-driven* views of CSR.

$F_s^{'}(x,S)>0$, $F_s^{''}(x,S)<0$, $\lim_{S\to 0} F_s^{'}(x,S)=\infty$ and $\lim_{S\to S_{MAX}} F_S^{'}(x,S)=0$. Finally, the unitary – constant - cost of CSR is τ.[20]

Proposition 3: An *enlightened* firm undertakes a positive level of CSR – defined as an investment which enhance the availability and/or quality of certain factors of production – in equilibrium, up to the level at which marginal costs equal marginal benefits. This level decreases in the firm's time-discount rate, i.e. a firm which has a more 'long run' perspective will undertake more CSR than one which cares exclusively of maximizing 'short-term' profits.

Proof*. The firm maximises the profits in equation (3), subject to the new law of motion given by:*

$$\dot{x} = F(x,S) - h(x,E) \qquad (9)$$

Now the control variables are two, S and E.[21] *The expression for the optimal level of utilization of the factor of production is the following:*

$$\rho = F_x^{'}(x,S) + \frac{\frac{w}{x^2}F(x,S)}{\overline{p}k - \frac{w}{x}} \qquad (10)$$

which has the same interpretation as before. The optimal level of CSR in equilibrium is determined by the following expression:

$$\tau = F_S^{'}(x,S)\left[\overline{p}k - \frac{w}{x}\right] \qquad (11)$$

which states that an enlightened firm undertakes CSR until its marginal cost, τ, equals the marginal benefit, which is given by the marginal effect of CSR on the rate of growth of the resource, multiplied by the (positive) difference between marginal revenues - $\overline{p}k$ - and marginal costs - $\frac{w}{x}$ - of obtaining one additional unit of the factor of production.

[20] It is worth highlighting that this is just the actual cost of undertaking CSR, but does not imply that CSR '*occurs at a cost for the firm*' - as it was for *externally-driven* views of CSR. In fact, we will see CSR actually decreases the marginal cost of producing the final good.

[21] See the Appendix for complete mathematical derivation.

The fact that $\lim_{S\to 0} F'(x,S) = \infty$*, ensures that, when the firm starts engaging in CSR, the right hand side of equation (11) is greater than the left hand side. Then, as the firm undertakes more CSR, the right hand side decreases because of decreasing marginal returns, until it hits the level* τ*. This ensures that equation (11) will be verified, and that the firm undertakes a positive level of CSR in equilibrium.*

In addition, we know from equation (10) that, the higher ρ*, the lower will be the stock x in equilibrium, and hence the lower the expression* $\left[\overline{p}k - \frac{w}{x}\right]$ *in equation (11). Hence, for the equality to be verified,* $F_S'(x,S)$ *will have to be higher, which implies that the equilibrium level of CSR will be lower.* **QED**

The other case (i.e. CSR increases the overall effectiveness of the production process) captures another common 'business' argument for CSR, which is that, for example, switching to more environmentally-friendly production technologies can lead to large increases in efficiency, energy saving, recycling of materials and reduction of waste. In terms of our model, we can assume that *k* - the overall efficiency of the production process CSR– is an increasing function of CSR, with decreasing marginal returns:

$$y = k(S)Ex \tag{12}$$

The optimal level of CSR in equilibrium is determined by the following expression:

$$\overline{p}k'(S)Ex = \tau \tag{13}$$

which states that an *enlightened* firm will undertake a positive level of CSR in equilibrium, up to the level at which the marginal cost τ equals the marginal benefit, given by the marginal increase in the efficiency of the production process - multiplied by the output's selling price.[22]

[22] The fact that $k'(S)$ is very high at the beginning and then decreases with the level of CSR, while τ is constant, ensures that the equality will be verified in equilibrium.

5. Conclusions and main limits of the model

Our model shows that an *enlightened* firm will voluntarily choose to reduce the level of utilization of certain factors of production, with respect to the level at which marginal private costs equal marginal revenues. The main idea driving our result is that *enlightened* firms take into account the 'dynamic stock externalities' from production, and their perception of the marginal cost of utilizing certain factors of production will be higher than the actual marginal cost. Since we assumed that these factors of production are also 'public goods', this implies that the firms' private marginal cost will now be closer to the one of the society, and the level of production closer to the social optimum (see Figure 1). In light of this, *enlightened* profit-maximising firms will look 'Socially Responsible' to the eyes of the society.[23]

It is important to stress that the reason why firms behave in such a way, is not altruistic motivation, but the fact that they realise that they can achieve higher profits by not utilizing the factors of production until marginal revenues equal marginal costs, but letting their stock (in the sense of their level and/or quality) grow and then utilize a higher and constant fraction for an infinite amount of time.[24] The profit-maximisation rationale is evident from the fact that an *enlightened* firm which only cares about 'short-term' profits (i.e. it has an infinite time-discount rate), will behave exactly as a firm which does not take into account how its choices affect the stock of the factors of production, and follow the marginal costs equal marginal revenues rule.

Our model also shows that *enlightened* firms will undertake a positive level of CSR, defined as investments which increase the availability and/or quality of the factors of production in the future. Again, the more they care about maximising 'long-term' profits as opposed to 'short-term' ones, the more of these practices will be undertaken in equilibrium.

One of the main questions to be addressed now is if these conclusions will hold if more than one firm has access to the same factor of production (i.e. the case of a 'common pool'). In the literature on renewable resources, this problem is known as the 'tragedy of the commons' (Dasgupta

[23] Interestingly, in the management literature CSR has been also defined as "Enlightened Self-interest" (Keim, 1978), or "Enlightened Value-Maximization" (Jensen, 2000)

[24] "When Mars and Cadbury talk about their cocoa supplies being sustainable, they mean it. Chocolate manufacturers are worried about how much cocoa will be available in a decade from now" ("Why Corporate Social Responsibility is a Survivor", *Financial Times*, 12th April 2009, p.13).

and Heal, 1979, Lehvari and Mirmam, 1980), because it often leads to the over-extraction – and sometimes the exhaustion - of the 'common' resource.

If we apply the same argument to our model, it would imply that firms will utilize the factor of production until the level at which marginal costs equal marginal benefits, because they are afraid that the other firms might do that anyway. This will lead to a Pareto-inefficient outcome – both for the firms and the society – with respect to when they all behave as *enlightened* firms.[25]

However, before analysing some of the possible solutions to this problem, it is worth highlighting that only some, among the factors of production which can be affected by CSR, have the characteristics of a 'common pool' resource. Typically, these are environmental resources, water, or employees' skills which can be easily transferred from one firm to another, etc. On the contrary, factors such as employees' motivation, goodwill, 'social licence to operate', support of the neighbouring communities, only to mention some, depend on the specific reputation - the 'CSR record' - of a firm, and are not accessible to other firms. Hence, in all these cases, the 'tragedy of the commons' argument will have a very limited impact.

In the cases in which the problem is relevant, one possible solution would be to give to one firm the exclusive access to the factor of production (e.g. a government licence). Such a policy would be in the interest of the society, avoiding the over-exploitation and inducing a greater investment in 'direct' CSR by firms.[26]

When it is not possible to give out licenses, under certain conditions it is still possible to achieve the Pareto-efficient outcome. In the context of a dynamic fishery game, Cave (1987) showed that, when players play a repeated game and are allowed to implement threats, every "credible, voluntary, collective agreement" to limit the exploitation is enforceable, provided that players are enough 'patient'. If we translate this argument to the context of CSR – assuming that the time-discount rate of *enlightened* firm is low enough - we can be optimistic that the Pareto-efficient

[25] This is Pareto-inefficient also for the firms' perspective, because for *enlightened* firms the choice to employ the factor up to the level at which marginal cost equal marginal benefits is NOT the profit-maximizing strategy.

[26] It should be noted that this exclusive access would have to be for an infinite time, or at least that the firms does know when it is the last period. Otherwise, if the firm expects the licence to expire at time t+n, it knows that it will not have access to the future benefits of being *enlightened*, thus at t+n-1 it will find optimal to employ the factor of production up to the level at which marginal costs equal marginal revenues. We will analyse this issue again later in the paper.

outcome can be sustained in equilibrium, provided that such game is played an infinite amount of times.

Another way in which the 'tragedy of the common' argument could have an impact in our model is to reduce the incentives for firms to undertake CSR. Because of the way we defined CSR, i.e. as an investment which increases the availability and/or quality of certain factors of production, then it is clear that when more than one firm has access to the same factor, some could free ride on the CSR programmes implemented by others, and benefit from the enhanced availability and/or quality of the resource without paying for it. This will reduce the private marginal benefits from CSR of the 'good' firms, leading them to reduce their provision.

There are many ways in which this problems could be – and often has been in practice - overcome. For example, a firm could invest in CSR programmes which target firm-specific factors of production and are not useful to any other firms or sectors, or could train their employees in specific skills which cannot be transferred without a cost to another firm or sector. Moreover, a firm could undertake CSR programmes which are highly differentiated and easily recognizable, such that that the benefits in terms of increased employees' motivation, goodwill and enhanced 'social licence to operate' cannot be reaped by other firms. Finally, when all of the above is not possible, or too costly to implement, firms should promote joint CSR programmes in certain areas, sharing the costs among all the firms that will benefit from them, in order to reduce the extent of possible free-riding by other firms.[27]

Finally, another key question is what would happen when a firm, which has access to a certain factor of production today, is not sure whether it will have access to it also in the future. The answer to this question can be summarised in one preposition, which completes the ones derived previously from the theoretical model.

[27] In the case of joint projects, we could expect to share the costs in proportion to the expected benefits from each projects, and that the final level of CSR at equilibrium will be equal to the Lindahl-Samuelson rule of marginal cost equal to the sum of marginal benefits for all firms. However, in reality most of the times the costs of a project are allocated taking into consideration the specific skills of each firms, in order to minimize total costs by maximizing contributions in kind by each firm, and allocating each task to the most efficient firm (see Balboni, Charles-Soverall and Levy, 2007).

Proposition 4: The degree to which an *enlightened* firm will behave according to the predictions of our model, with respect to a certain factor of production, depends on its perception of the possibility to have access to that factor also in the future.[28]

In fact, from a theoretical perspective, the condition for a firm to behave as *enlightened* is that it has an infinite future access to the relevant factor of production, or at least that it attaches a positive – and large enough – probability to have access to it for any period in the future. In fact, as this probability goes to zero, it would be 'as if' the firm had an infinite time-discount rate, and it would find it optimal to utilize the factor up to the level where marginal cost equal marginal benefits.

This will also affect the extent to which a firm engage in CSR, because it might be reluctant to undertake an investment which increases the level and/or quality of a factor of production, without being certain that it will have access to that factor in the future.

This issue is relevant for a much wider range of factors of production than the ones affected by the 'common pool' problem. In particular, the firm's choices will be affected every time a CSR investment does not yield benefits for the firm in general (e.g. increased reputation, goodwill, 'social licence to operate', etc.), but only on the specific factor to which the investment was directed to. (i.e. an employer, a community, a government, etc.). Some examples might be the choice to train the employees, when the firm is afraid that one day they might leave and re-employ their skills somewhere else, or the choice to undertake a joint CSR project with the government of a particular country, when the firm's licence to operate is about to expire.

Fortunately, empirical evidence supports the idea that the probability that a firm - which has access to one resource today - will also have it in the future, is not exogenous, but actually depends on the firm's behaviour 'towards' the resource in the past, and on the expectations of its behaviour in the future. For example, many studies showed that firms which are perceived to be 'Socially Responsible' are better able to attract and retain the and most productive and most motivated employees (Brekke and Nyborg, 2005; Collier and Esteban, 2007). Similarly, the probability that the

[28] If we combine the problem of future access to the 'common pool' one, then the condition to enforce collusion would be that all the firms, which currently have access to the factor of production, should have a positive probability of accessing the factor for every period in the future; or at least that all the firms should attach a non-zero probability to his, and all the other firms' future access to the factor. Otherwise, collusion will not be sustainable and, by backward induction, firms will employ the factor of production up to the level at which marginal costs equal marginal benefits, already in the first period.

government will grant a licence to operate to a firm depends on the firm's reputation and on the CSR programmes it commits to undertake in the country.

These empirical regularities are good news for our model, because they actually reinforce the mechanisms which lead firms to behave as *enlightened* and undertake CSR.

Nonetheless, further empirical research is needed, to understand under which conditions firms actually behave as *enlightened*, and to which extent they do so, depending on the different characteristics of the relevant factors of production.

In addition, other studies might be tailored to test some of the specific implications of our model. For example, our model seems to suggest that long-term work contracts or long-term government licences might induce firms to undertake more CSR, and it would be interesting to know to which extent the data match these predictions.

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APPENDIX. MATHEMATICAL DERIVATIONS

Derivation of Propositions 1 and 2.

We set up the Current Value Hamiltonian, given by:

$$H = \overline{p}kEx - wE + \lambda\left[F\left(x\right) - Ex\right] \tag{1}$$

To solve this, we set the derivative with respect to the control variable E equal to 0:

$$\frac{\partial H}{\partial E} = 0 \Rightarrow \overline{p}kx - w = \lambda x \tag{2}$$

and the derivative with respect to the state variable x equal to $\rho\lambda - \dot{\lambda}$*, where* $\dot{\lambda}$ *is the derivative of the Lagrange multiplier with respect to time:*

$$\frac{\partial H}{\partial x} = \rho\lambda - \dot{\lambda} \Rightarrow pkE + \lambda\left[F^{'}\left(x\right) - E\right] = \rho\lambda - \dot{\lambda} \tag{3}$$

From equation we can obtain an expression for λ*:*

$$\lambda = \overline{p}k - \frac{w}{x} \tag{4}$$

If we derive this expression with respect to time, we obtain:

$$\dot{\lambda} = -\frac{w}{x^2}\dot{x} \tag{5}$$

Plugging equations (4) *and* (5) *into the right hand side of equation* (3)*, we get the fundamental expression for the optimal utilization of the factor of production:*

$$\rho = F^{'}\left(x\right) + \frac{\frac{w}{x}E + \frac{w}{x^2}\dot{x}}{\overline{p}k - \frac{w}{x}} \tag{6}$$

This expression can be rewritten in the following way, to eliminate the control variable E:

$$\rho = F^{'}\left(x\right) + \frac{\frac{w}{x^2}F\left(x\right)}{\overline{p}k - \frac{w}{x}} \tag{7}$$

Derivation of Proposition 3.

The firm maximises the profits in equation (3)*, under the new constraint that:*

$$\dot{x} = F\left(x,S\right) - h\left(x,E\right) \tag{8}$$

We have now two control variables, S and E. The Current-Value Hamiltonian for this problem is:

$$H = \overline{p}kEx - wE - \tau S + \lambda\left[F\left(x,S\right) - Ex\right] \tag{9}$$

To solve this, we set:

$$\frac{\partial H}{\partial E} = 0 \Rightarrow \overline{p}kx - w = \lambda x \tag{10}$$

$$\frac{\partial H}{\partial S} = 0 \Rightarrow \tau = \lambda F_S^{'}\left(x,S\right) \tag{11}$$

$$\frac{\partial H}{\partial x} = \rho\lambda - \dot{\lambda} \Rightarrow pkE + \lambda\left[F_x^{'}\left(x,S\right) - E\right] = \rho\lambda - \dot{\lambda} \tag{12}$$

The two conditions (10) *and* (11) *are independent, and can be solved separately, which allows for a closed form solution. The expression for the optimal level of utilization of the input is the following:*

$$\rho = F_x^{'}\left(x,S\right) + \frac{\frac{w}{x^2}F\left(x,S\right)}{\overline{p}k - \frac{w}{x}} \tag{13}$$

By plugging the value of λ *from equation* (10) *into equation (11), we obtain the equation which determines the optimal level of CSR in equilibrium:*

$$\tau = F_S^{'}\left(x,S\right)\left[\overline{p}k - \frac{w}{x}\right] \tag{14}$$